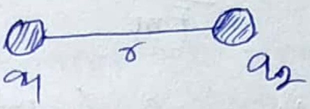
—: Electrostatics :—

Kamal 07/02/18 ①

Coulomb's law in electrostatics: The magnitude of force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of distance between them and it acts along the line joining to the point charges.



In mathematically, $F \propto q_1 q_2$

$\propto 1/r^2$

$\therefore F \propto \frac{q_1 q_2}{r^2} \Rightarrow F = C \frac{q_1 q_2}{r^2}$, where C is coulomb's constant.

and the value of C depends on medium. Value of C is

$C = 9 \times 10^9 \ N\text{-}m^2/C^2$ in free space.

and permittivity which implies characteristic of the medium. where ϵ is the permittivity.

Thus. $C = \frac{1}{4\pi\epsilon}$, where ϵ is the permittivity.

Now, value of $\epsilon_0 = 8.85 \times 10^{-12} \ C^2/N\text{-}m^2$; where ϵ_0 is the permittivity in free space.

$\therefore \boxed{F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}}$ is the coulomb's law in electrostatics.

In vectorically, $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r} = \vec{F}}$

Unit charge:- If $q_1 = q_2 = q$ (say) & $r = 1\,m$. then,

$F = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ N$; for free space.

i.e. when two identical charges are placed in air separated by a distance 1 m and a force of repulsion 9×10^9 N is experienced, then the charges are separately called unit charge.

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$\therefore F_0 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$ — ① ; for free space.

$F_m = \frac{1}{4\pi\epsilon_m}\frac{q_1 q_2}{r^2}$ — ② ; for any medium.

Now, $\frac{F_0}{F_m} = \frac{\epsilon_m}{\epsilon_0} = \epsilon_r$ | $\frac{F_m}{F_0} = \frac{\epsilon_0}{\epsilon_m} \Rightarrow F_m = \frac{\epsilon_0}{\epsilon_m} F_0$

since, ϵ_r (relative permittivity) $= \frac{\epsilon_m}{\epsilon_0}$ $\boxed{\therefore \epsilon_m = \epsilon_r \epsilon_0}$

ϵ_r is 1 for free space & no unit.

$\therefore F_m = \frac{F_0}{\epsilon_r} \Rightarrow \boxed{\epsilon_r = \frac{F_0}{F_m}}$. relative permittivity is decrease for any medium.

Now, again, $\frac{F_0}{F_m} = K$ [where K is dielectric const]

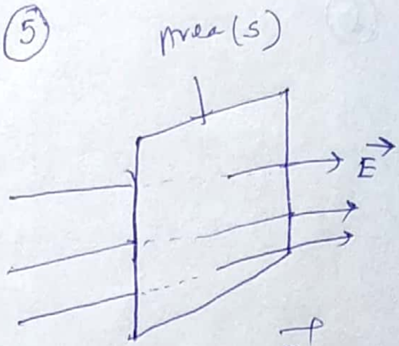
$\boxed{\therefore \epsilon_r = K}$ [$\because$ relative permittivity is equal to dielectric constant].

Now, coulombs law, $F_0 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$

$F_m = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1 q_2}{r^2}$

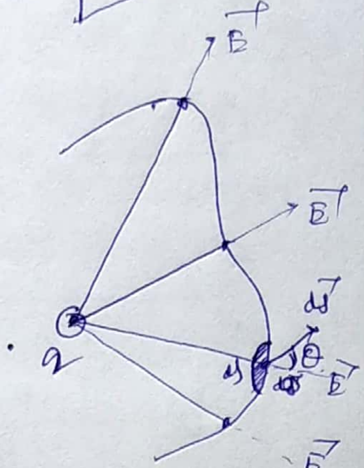
$\therefore$ Finally, $\boxed{F = \frac{1}{4\pi\epsilon_0 K}\frac{q_1 q_2}{r^2}}$

Electric flux :-



Electric flux linked with any surface is defind as total number of electric field line pass through that area.

Consider a surface area S. Let electric field lines representing electric field $\vec{E}$ pass through the plane of area S perpendicular to the plane of the surface. Then the electric flux through the surface of area S is given by,



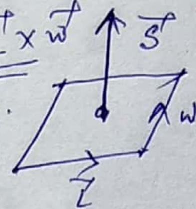
$$\boxed{\phi = E \cdot S}$$

scalar vector vector.

Now, consider the surface is made of small elements each of area vector $d\vec{s}$. Let θ be the angle made by the electric field $\vec{E}$ with the area vector $d\vec{s}$ of the surface element. Then, the electric flux through the surface element is given by,

Area $(\vec{S}) = \vec{L} \times \vec{w}$

$S = LW \sin\theta$



$$d\phi = \vec{E}\, ds \cos\theta = \vec{E} \cdot d\vec{s}$$

$\therefore$ Total flux through the surface is given by,

$$\int d\phi = \int_S E\, ds \cos\theta = \boxed{\phi = \int_S \vec{E} \cdot d\vec{s}}$$

Special case

(i) If $\theta = 0^\circ$ i.e, $\vec{E}$ is perpendicular to the plane of the surface, then electric flux through the surface is given by,

$$\phi = \int E\, ds \cos 0^\circ = \int E\, ds = ES.$$

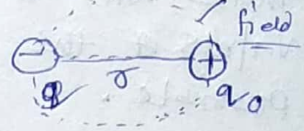
(ii) If $\theta = 90^\circ$, i.e, $\vec{E}$ is parallel to the plane of the surface, the electric flux is given by,

$$\phi = \int E\, ds \cos 90^\circ = 0.$$

S.I unit of electric flux $(\phi) = N\text{-}m^2/C$

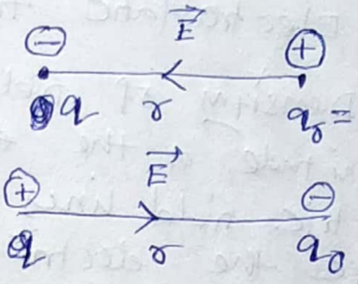
Electrostatic field :

Electrostatic field is the space sorrounding a point charge at every point of which the influence (attraction or repulsion) is enperienced.



Intensity of electric field :

The free experienced by a point unit positive charge placed at any point in the field of a charge is called intensity of electric field at that point.



From coulomb's law this free is $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} \hat{r} = q_0 \vec{E}$.

where, $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}}$ $[\because q_0 = 1]$

If q is positive $\vec{E}$ is directed away from q. on the other hand if q is negative, then $\vec{E}$ is directed towards q.

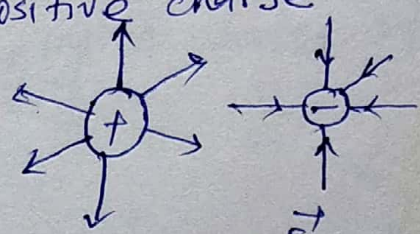
Remember $\vec{F} = q_0 \vec{E} \Rightarrow \vec{E} = \frac{\vec{F}}{q_0}$; unit of $\vec{E}$ is N/C.

Electric field lines :

Electric field lines are curved or straight imaginary line in the electric field such that the tangent at any point on the field line gives the direction of the electric field at that point.

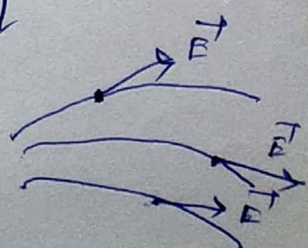
Properties of Electric field lines :

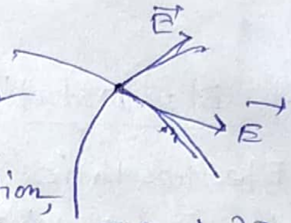
1. The electric field lines begin from positive charge and terminate or end on negative charge.



2. Electric field lines are imaginary.

3. The tangent at any point on an electric field line gives the direction of the electric field at that point.

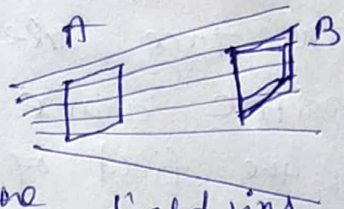




4. Two electric field lines cannot cross each other. because of if two electric field line cross each other, then at the point of intersection, there will be two tangents. It means that there are two values of the electric field at that point, which is not possible.

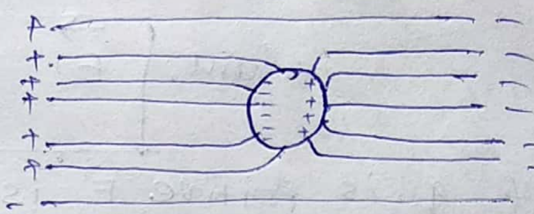
5. Electrostatic field lines do not form closed loops.

6. Density of field lines represents the magnitude of the electric field. Thus, the electric field lines are closer (crowded), where the electric field is stronger and the field lines spread out, where the electric field is weaker.

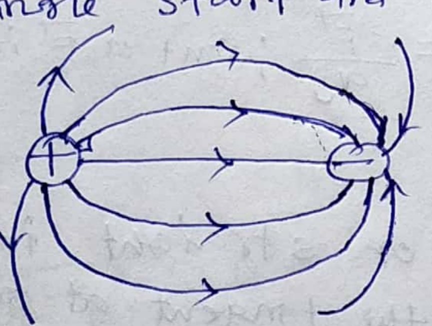


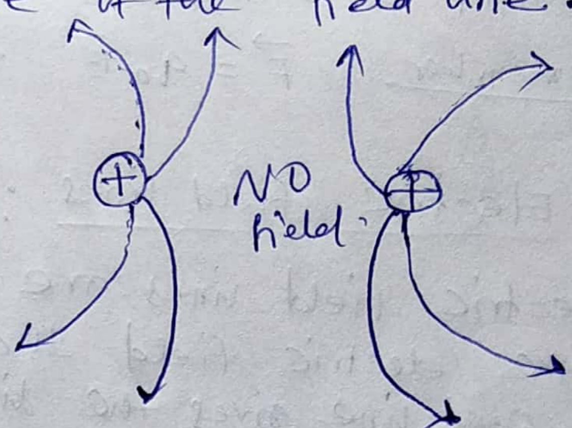
7. In a region where is no electric field, lines are absent. This is why inside a conductor there, cannot be any electric field line.

Metal or conductor



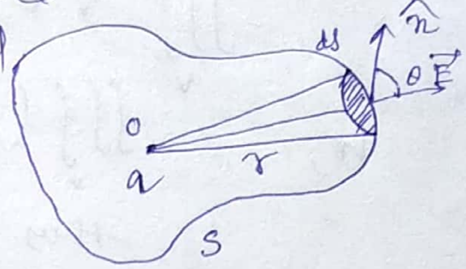
8. right angle start and terminate of the field line.





State Gauss's theorem in electrostatics :

Total electric flux over any closed surface is equal to $1/\epsilon_0$ times the net charge enclosed by the surface.

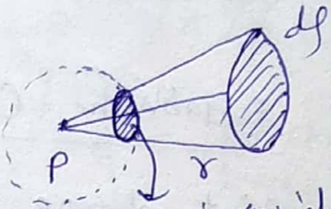


i.e, $$\oint_S \vec{E}\cdot\hat{n}\,d\vec{s} = q/\epsilon_0$$

Proof: Let consider a point charge q at O.

electric field at a distance r from O,

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$



flux through the elementary area,

$$d\phi = \vec{E}\cdot\hat{n}\,d\vec{s}$$

$d\omega$ is solid angle of ds at P inside the cone.

$$\boxed{d\omega = \frac{ds}{r^2}}$$

Now, total flux over whole area S,

$$\oint_S \vec{E}\cdot\hat{n}\,d\vec{s} = \oint_S \frac{q}{4\pi\epsilon_0 r^2}\hat{r}\cdot\hat{n}\,d\vec{s}$$

$$= \frac{q}{4\pi\epsilon_0}\oint_S \frac{ds\cos\theta}{r^2}$$

$$d\omega = \frac{ds\cos\theta}{r^2}$$

$$= \frac{q}{4\pi\epsilon_0}\oint_S d\omega$$

$d\omega$ is the solid angle subtended by ds at O.

$$= \frac{q}{\cancel{4\pi}\epsilon_0}\times\cancel{4\pi} = q/\epsilon_0.$$

$$\oint_S d\omega = 4\pi$$

$$\boxed{\therefore \oint_S \vec{E}\cdot\hat{n}\,d\vec{s} = q/\epsilon_0.}$$ Proof.

Differential form of Gauss's theorem

we have, $$\oint_S \vec{E}\cdot\hat{n}\,ds = \frac{q}{\epsilon_0}$$

Now, $$\oint_S \vec{E}\cdot\hat{n}\,ds = \frac{1}{\epsilon_0}\iiint_v \rho\,dv \quad \text{—①}$$

where, ρ is the volume ~~of charge~~ density of charge and dv is the arbitrary volume.

According to divergence theorem.

$$\oint_S \vec{E}\cdot\hat{n}\,ds = \iiint_v \vec{\nabla}\cdot\vec{E}\,dv \quad \text{—②}$$

From eqⁿ 1 & 2

$$\iiint_V \vec{\nabla}\cdot\vec{E}\, dv = \frac{1}{\epsilon_0}\iiint_V \rho\, dv$$

or,

$$\iiint_V \left(\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\epsilon_0}\right) dv = 0$$

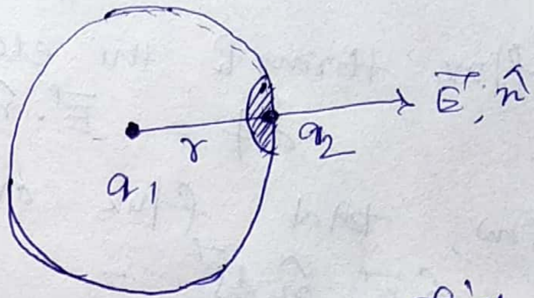
thus $dv \neq 0$. $\therefore \vec{\nabla}\cdot\vec{E} - \frac{\rho}{\epsilon_0} = 0$

$$\boxed{\therefore \vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0.}$$

This is the differential form of Gauss theorem.

Establish Coulomb's law from Gauss's theorem:

A point charge q_1 is placed at any point in free space. We have to calculate the ~~point charge~~ electric field at a distance r from the point charge, where another charge q_2 is placed.



We take a surface of sphere of radius r, which is called Gaussian surface.

According to Gauss's theorem,

$$\oint_s \vec{E}\cdot\hat{n}\, ds = \frac{q_1}{\epsilon_0} \qquad \vec{E}\cdot\hat{n} = E$$

$$E\oint_s ds = q/\epsilon_0$$

$$\Rightarrow E\times 4\pi r^2 = q_1/\epsilon_0.$$

$$\Rightarrow E = \frac{q_1}{4\pi\epsilon_0 r^2}.$$

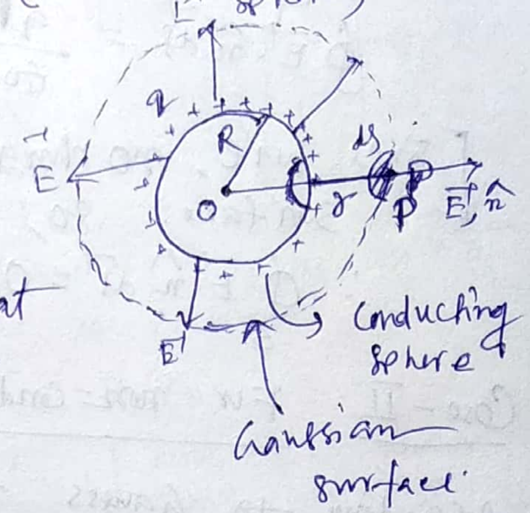
Now, force on q_2 is $F = q_2 E = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$

$$\boxed{\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}}$$

Application of Gauss's theorem :-

① Electric field due to uniformly charged spherical shell :- (conductor and non-conducting sphere)

Let consider a conducting sphere of radius R. Let charge q distributed surface on the sphere.



(a) At a point outside of the sphere $(r > R)$

We want to calculate electric field $(\vec{E})$ at a distace r from O. We consider a Gaussion surface.

According to Gauss's theorem,

$$\oint \vec{E}\cdot\hat{n}\, ds = q/\epsilon_0$$

$$\Rightarrow E \oint ds = q/\epsilon_0 \qquad [\theta = 0^\circ,\ \cos\theta^\circ = 1]$$

$$\Rightarrow E \times 4\pi r^2 = q/\epsilon_0 \qquad [\oint ds = 4\pi r^2]$$

or, $$\boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$$

In terms of charge density $(\rho) = \frac{\text{total charge}}{\text{volume}} = \frac{q}{\frac{4}{3}\pi R^3}$

$$\therefore q = \frac{4}{3}\pi R^3 \rho \qquad [\text{where } \rho \text{ is the volume density of charge}]$$

Now, $$\vec{E} = \frac{\frac{4}{3}\pi R^3 \rho}{4\pi\epsilon_0 r^2}\hat{r} = \boxed{\frac{\rho R^3}{3\epsilon_0 r^2}\hat{r} = \vec{E}}$$ field in terms of charge density

(b) At a point on the surface of the sphere $(r = R)$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 R^2}\hat{r} = \frac{\frac{4}{3}\pi R^3 \rho}{4\pi\epsilon_0 R^2}\hat{r}$$

$$\boxed{\vec{E} = \frac{\rho R}{3\epsilon_0}\hat{r}}$$

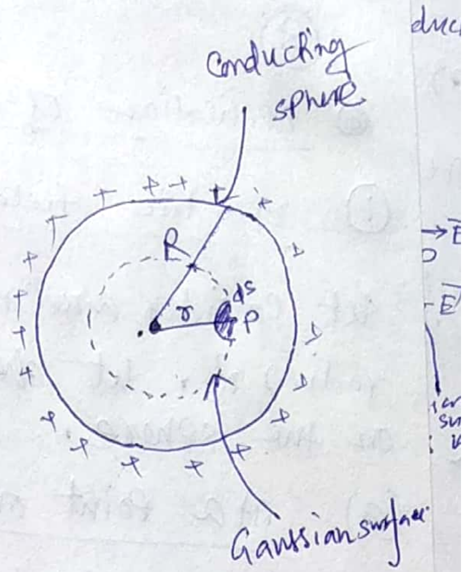
⑨ ⑫

(a) At a point inside of the sphere ($r < R$)

Case – I [for conducting sphere]

According to Gauss's theorem

$$\oint \vec{E} \cdot \hat{n}\, d\vec{s} = \frac{q'}{\epsilon_0}$$ [q' is the charge outside of the sphere]



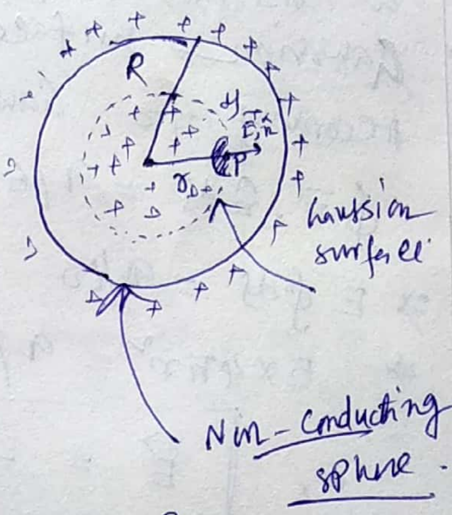
[Since here, no charge inside the Gaussian surface. So, $q = 0$

$\therefore \oint_s \vec{E} \cdot \hat{n}\, d\vec{s} = 0$ or, $\boxed{\vec{E} = 0}$

Case – II For non-conducting sphere.

According to Gauss's theorem,

$$\oint_s \vec{E} \cdot \hat{n}\, d\vec{s} = \frac{q'}{\epsilon_0}$$ [where, q' is the charge inside the Gaussian surface]



or, $E \oint_s ds = \frac{q'}{\epsilon_0}$ [$\theta = 0°$, $\cos 0° = 1$]

or, $E \times 4\pi r^2 = \frac{q'}{\epsilon_0}$

$$\boxed{\text{or, } \vec{E} = \frac{q'}{4\pi\epsilon_0 r^2}\hat{r}}$$

In terms ρ, $q' = \frac{4}{3}\pi r^3 \rho$ $\therefore \vec{E} = \frac{\frac{4}{3}\pi r^3 \rho}{4\pi\epsilon_0 r^2}\hat{r}$

$$\boxed{\vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r}}$$ [in terms of charge density ρ]

Now, $\frac{q'}{q} = \frac{\frac{4}{3}\pi r^3 \rho}{\frac{4}{3}\pi R^3 \rho} = \frac{r^3}{R^3}$ $\therefore q' = \frac{r^3}{R^3} q$

Now, $\vec{F} = \frac{r^3 q}{4\pi\epsilon_0 r^2 R^3} = \frac{rq}{4\pi\epsilon_0 R^3}\hat{r}$

$$\boxed{\therefore \vec{E} = \frac{qr}{4\pi\epsilon_0 R^3}\hat{r}}$$ [in terms of total charge]

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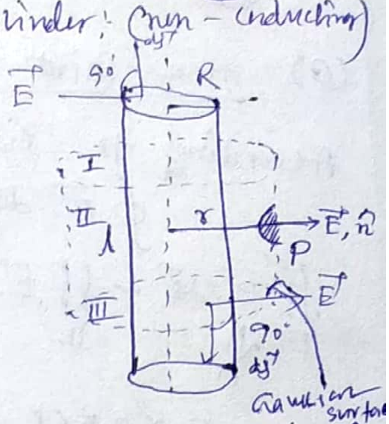
- Electric field due to a uniformly charged cylinder: (non - conducting)



(a) At a point outside of the cylinder
(b) At a point on the surface of the cylinder
(c) At a point inside of the cyliner.

(a) At a point outside of the cylinder ($r > R$)

Let us consider a cylinder of length l and radius R. We calculate electric field at P at a distance r from the axis of the cylinder. We consider a Gaussian surface of radius r.

According to Gauss's theorem, $\oint_s \vec{E} \cdot \hat{n}\, ds = q/\epsilon_0$

Now, $\underbrace{\iint_{I} \vec{E} \cdot \vec{ds}}_{0} + \iint_{II} \vec{E} \cdot \vec{ds} + \underbrace{\iiint_{III} \vec{E} \cdot \vec{ds}}_{0} = \frac{\lambda l}{\epsilon_0}$

[here, λ is linear charge density $\lambda = q/l$ $\boxed{\therefore q = \lambda l}$]

for surface I and III, angle bet^n $\vec{E}$ & $\vec{ds}$ is 90°.

So $\vec{E} \cdot \vec{ds} = 0$

$$\therefore E \iint_{II} ds = \frac{\lambda l}{\epsilon_0}$$

[$\because \vec{E}\, ds \cos 0° = E\, ds$]

$$\Rightarrow E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

[surface area of the cylinder $= 2\pi r l$]

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$$

If ρ represents the volume charge density, then the relation between λ and ρ is as follows, $\lambda l = \pi R^2 l \rho \Rightarrow \lambda = \pi R^2 \rho$

$$\boxed{\vec{E} = \frac{\pi R^2 \rho}{2\pi \epsilon_0 r} = \frac{R^2 \rho}{2\epsilon_0 r} \hat{r}}$$

(b) At the surface on the sphere ($r = R$)

$$\boxed{\vec{E} = \frac{R^2 \rho}{2\epsilon_0 R} \hat{r} = \frac{R\rho}{2\epsilon_0} \hat{r}}$$

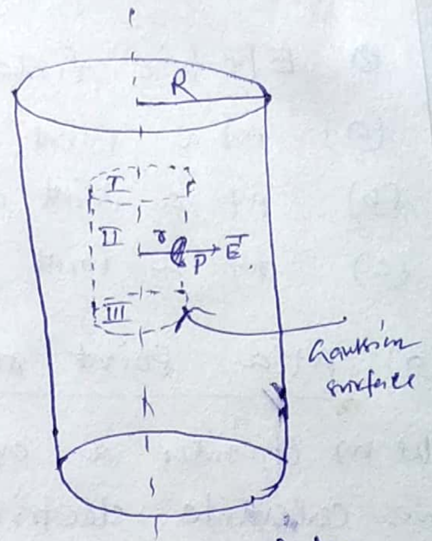
(14)

(a) At a point inside of the cylinder $(r<R)$

According to Gauss's theorem,

$$\oint \vec{E}\cdot d\vec{s} = \frac{q'}{\epsilon_0}$$ [q' → charge enclosed by the cylinder]

$$\underbrace{\iint_{I} \vec{E}\cdot d\vec{s}}_{0} + \iint_{II} \vec{E}\cdot d\vec{s} + \underbrace{\iint_{III} \vec{E}\cdot d\vec{s}}_{0} = \frac{\pi r^2 l \rho}{\epsilon_0}$$



$$\Rightarrow E \times 2\cancel{\pi} r \cancel{l} = \frac{\cancel{\pi} r^{\cancel{2}} \cancel{l} \rho}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho r}{2\epsilon_0}\hat{r}}$$

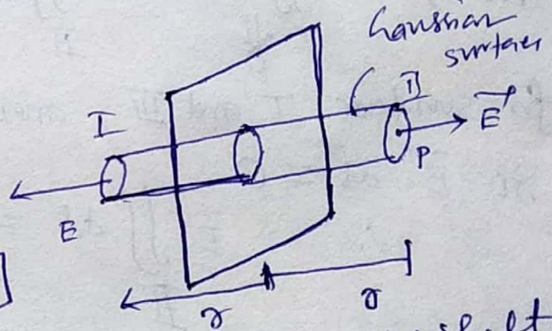
[volume of the cylinder $= \pi r^2 l$]

Electric field due to a uniformly charged plane sheet:

Consider a plane sheet having surface charge density σ.

$$\sigma = q/s$$

$\therefore q = \sigma s$ [charge per unit area]



To calculate electric field at P at a distance r from the sheet.

Consider a Gaussian surface.

According to Gauss's theorem, $\oint_S \vec{E}\cdot d\vec{s} = q/\epsilon_0$

$$\iint_{I} \vec{E}\cdot d\vec{s} + \iint_{II} \vec{E}\cdot d\vec{s} = \frac{\sigma s}{\epsilon_0}$$

$$\Rightarrow 2ES = \frac{\sigma s}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

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